Calculus II — Homework 0420 — Name:

Dr. Paul L. Bailey — Monday, April 19, 2021

Due Tuesday, April 20.

Proposition 1. (Geometric Series Test)
A geometric series *is a series of the form* $\sum ar^n$. *The series converges if and only if* $|r| < 1$. *If* $|r| < 1$, *then*

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

Note that if $\sum_{n=0}^{\infty} a_n$ converges and k is a positive integer, then $\sum_{n=k}^{\infty} a_n = \sum_{n=k}^{\infty} - \sum_{n=0}^{k-1} r^n$.

Problem 1. Find the values of the following series.

(a) $\sum_{n=0}^{\infty} 2 \cdot (0.6)^n$

(b) $\sum_{n=2}^{\infty} \dfrac{2^n + 3^n}{5^n}$

Proposition 2. (p-Series Test)
The series $\sum \dfrac{1}{n^p}$ *converges if and only if* $p > 1$.

Proposition 3. (Direct Comparison Test)
Suppose that $0 \leq a_n \leq b_n$ *for all* $n \in \mathbb{N}$. *Then*

- *If* $\sum b_n$ *converges, then* $\sum a_n$ *converges.*
- *If* $\sum a_n$ *diverges, then* $\sum b_n$ *diverges.*

Problem 2. Write a complete sentence which uses the Direct Comparison Test to analyze whether or not the given series converges.

(a) $\sum_{n=2}^{\infty} \dfrac{1}{n + 2^n}$

(b) $\sum_{n=2}^{\infty} \dfrac{1}{n - n^2}$

Proposition 4. (Ratio Test)
Let $a_n \leq 0$ *for all* n. *Let* $\rho = \lim \frac{a_{n+1}}{a_n}$.

- *If* $\rho < 1$, *then* $\sum a_n$ *converges.*
- *If* $\rho > 1$, *then* $\sum a_n$ *diverges.*
- *If* $\rho = 1$, *then the test is inconclusive.*

Problem 3. Use the Ratio Test to determine if the following series converge. Write your findings in a complete sentence which justifies your conclusion.

(a) $\sum \frac{2^n \cdot 3^n}{n^n}$

(b) $\sum \frac{3^n}{n!}$

Problem 4. (Pick it)
Determine if $\sum \frac{2^n \cdot 3^n}{n^n}$ converges. Justify your answer in complete sentences.